Suppose a context $\mathbf{K} = (G, M, I)$ and a new column $\mathbf{C} = (G, \{n\}, J)$ with $n \notin M$ are given. Then we describe in terms of structure what happens with the labeled concept lattice diagram upon insertion of $\mathbf{C}$ into $\mathbf{K}$ and removal of $\mathbf{C}$ from $\mathbf{K}|\mathbf{C}$. For this purpose the notion of formal concepts is extended to a quadrupel $(A, B, A_\lambda, B_\lambda)$ with A and B being the extent and intent as usual, and furthermore A_λ and B_λ contain all object and attribute labels. Such a quadrupel is called concept node. The concept nodes of $\mathbf{K}|\mathbf{C}$ can be computed from those of $\mathbf{K}$ and vice versa, by classifying them and then updating them by the following mappings.

(OLD) Each old concept of $\mathbf{K}$ *w.r.t.* $\mathbf{C}$ is an old concept of $\mathbf{K}|\mathbf{C}$ as well, and conversely every old concept of $\mathbf{K}|\mathbf{C}$ is an old concept of $\mathbf{K}$ *w.r.t.* $\mathbf{C}$, by the bijection

$$\mathrm{old}_{\mathbf{C}}\colon \quad \mathfrak{N}^{\mathbf{C}}_{\mathrm{old}}(\mathbf{K}) \hookrightarrow\!\!\!\!\rightarrow \mathfrak{N}_{\mathrm{old}}(\mathbf{K}|\mathbf{C})$$

$$(A, B, A_\lambda, B_\lambda) \mapsto \begin{cases} (A, B, A_\lambda, B_\lambda) & \text{if } (A,B) \notin \mathfrak{B}^{\mathbf{C}}_{\mathrm{gen}}(\mathbf{K}) \\ (A, B, A_\lambda \setminus n^J, B_\lambda) & \text{if } (A,B) \in \mathfrak{B}^{\mathbf{C}}_{\mathrm{gen}}(\mathbf{K}) \end{cases}$$

$$\left.\begin{array}{ll} (A, B, A_\lambda, B_\lambda) & \text{fi } (A,B) \notin \mathfrak{B}_{\mathrm{gen}}(\mathbf{K}|\mathbf{C}) \\ (A, B, A_\lambda \,\dot\cup\, C_\lambda, B_\lambda) & \text{fi } (A,B) \in \mathfrak{B}_{\mathrm{gen}}(\mathbf{K}|\mathbf{C}) \text{ and } (C,D) = \mathrm{new}_{\mathbf{C}}(\mathrm{old}^{-1}_{\mathbf{C}}(A,B)) \end{array}\right\} \mapsfrom (A, B, A_\lambda, B_\lambda)$$

with $\mathfrak{B}^{\mathbf{C}}_{\mathrm{old}}(\mathbf{K}) = \{(A,B) \in \mathfrak{B}(\mathbf{K}) \mid A \not\subseteq n^J\}$ and $\mathfrak{B}_{\mathrm{old}}(\mathbf{K}|\mathbf{C}) = \{(A,B) \in \mathfrak{B}(\mathbf{K}|\mathbf{C}) \mid n \notin B\}$.

(VAR) Every varying concept of $\mathbf{K}$ *w.r.t.* $\mathbf{C}$ is mapped to a varied concept of $\mathbf{K}|\mathbf{C}$ by adding the new attribute n to the intent, and reversely each varied concept of $\mathbf{K}|\mathbf{C}$ become a varying concept of $\mathbf{K}$ *w.r.t.* $\mathbf{C}$ be removing n from its intent. This is due to the bijection

$$\mathrm{var}_{\mathbf{C}}\colon \quad \begin{array}{rcl} \mathfrak{N}^{\mathbf{C}}_{\mathrm{var}}(\mathbf{K}) & \hookrightarrow\!\!\!\!\rightarrow & \mathfrak{N}_{\mathrm{var}}(\mathbf{K}|\mathbf{C}) \\ (A, B, A_\lambda, B_\lambda) & \mapsto & (A, B \,\dot\cup\, \{n\}, A_\lambda, B_\lambda) \\ (A, B \setminus \{n\}, A_\lambda, B_\lambda) & \mapsfrom & (A, B, A_\lambda, B_\lambda) \end{array}$$

with $\mathfrak{B}^{\mathbf{C}}_{\mathrm{var}}(\mathbf{K}) = \{(A,B) \in \mathfrak{B}(\mathbf{K}) \mid A \subseteq n^J\}$ and $\mathfrak{B}_{\mathrm{var}}(\mathbf{K}|\mathbf{C}) = \{(A,B) \in \mathfrak{B}(\mathbf{K}|\mathbf{C}) \mid n \in B \text{ and } (B \setminus \{n\})^I = A\}$.

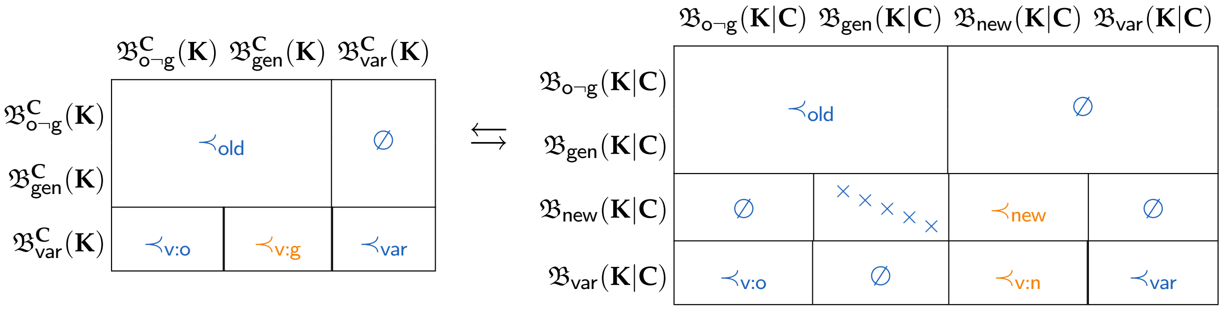
(GEN/NEW) Each new concept of $\mathbf{K}|\mathbf{C}$ can be constructed from a unique generating concept of $\mathbf{K}$ *w.r.t.* $\mathbf{C}$ by intersecting the extent with the new attribute extent n^J and adding the new attribute n to the intent. The map

$$\mathrm{new}_{\mathbf{C}}\colon \quad \mathfrak{N}^{\mathbf{C}}_{\mathrm{gen}}(\mathbf{K}) \hookrightarrow\!\!\!\!\rightarrow \mathfrak{N}_{\mathrm{new}}(\mathbf{K}|\mathbf{C})$$

$$(A, B, A_\lambda, B_\lambda) \mapsto \begin{cases} (A \cap n^J, B \,\dot\cup\, \{n\}, A_\lambda \cap n^J, B_\lambda) & \text{if } n^J \not\subset A \text{ iff } (A,B) \neq \top^{\mathbf{C}}_{\mathrm{gen}} \\ (n^J, B \,\dot\cup\, \{n\}, A_\lambda \cap n^J, B_\lambda \,\dot\cup\, \{n\}) & \text{if } n^J \subset A \text{ iff } (A,B) = \top^{\mathbf{C}}_{\mathrm{gen}} \end{cases}$$

is a bijection, with its domain $\mathfrak{B}^{\mathbf{C}}_{\mathrm{gen}}(\mathbf{K}) = \{(A,B) \in \mathfrak{B}(\mathbf{K}) \mid A \not\subseteq n^J \text{ and } (A \cap n^J)^I = B\}$ and range $\mathfrak{B}_{\mathrm{new}}(\mathbf{K}|\mathbf{C}) = \{(A,B) \in \mathfrak{B}(\mathbf{K}|\mathbf{C}) \mid n \in B \text{ and } (B \setminus \{n\})^I \neq A\}$. Although not needed for practical purposes, also the generator concept nodes can be computed from the new concept nodes. Upon removal of the column $\mathbf{C}$ these new concept nodes are rather removed from the concept diagram.

We are able to completely describe the neighborhood relation of $\mathbf{K}|\mathbf{C}$ by means of the cover relation of $\mathbf{K}$ and vice versa. Especially when thinking of cover relations as binary relations encoded by binary matrices, the cover relations can be determined from each other by simply copying some parts, deleting some parts, and computing few parts. For this purpose the cover relation of $\underline{\mathfrak{B}}(\mathbf{K})$ and also the cover relation of $\underline{\mathfrak{B}}(\mathbf{K}|\mathbf{C})$ are split up in components



and the unknown parts can be computed via

$$\forall_{\substack{(A,B)\in\mathfrak{B}_{\mathsf{var}}(\mathbf{K}|\mathbf{C})\\(C,D)\in\mathfrak{B}_{\mathsf{new}}(\mathbf{K}|\mathbf{C})}} \mathsf{var}_{\mathbf{C}}^{-1}(A,B) \prec_{\mathsf{v:g}} \mathsf{gen}_{\mathbf{C}}(C,D) \Leftrightarrow \begin{cases} (A,B) \prec_{\mathsf{v:n}} (C,D) \text{ and} \\ \nexists_{(X,Y)\in\mathfrak{B}_{\mathsf{o}\neg\mathsf{g}}(\mathbf{K}|\mathbf{C})} \quad (A,B) < (X,Y) < \mathsf{old}_{\mathbf{C}}\mathsf{gen}_{\mathbf{C}}(C,D) \end{cases}$$

$$\forall_{(A,B),(C,D)\in\mathfrak{B}^{\mathbf{C}}_{\mathsf{gen}}(\mathbf{K})} \mathsf{new}_{\mathbf{C}}(A,B) \prec_{\mathsf{new}} \mathsf{new}_{\mathbf{C}}(C,D) \Leftrightarrow \begin{cases} (A,B) < (C,D) \text{ and} \\ \nexists_{(X,Y)\in\mathfrak{B}^{\mathbf{C}}_{\mathsf{gen}}(\mathbf{K})} \quad (A,B) < (X,Y) < (C,D) \end{cases}$$

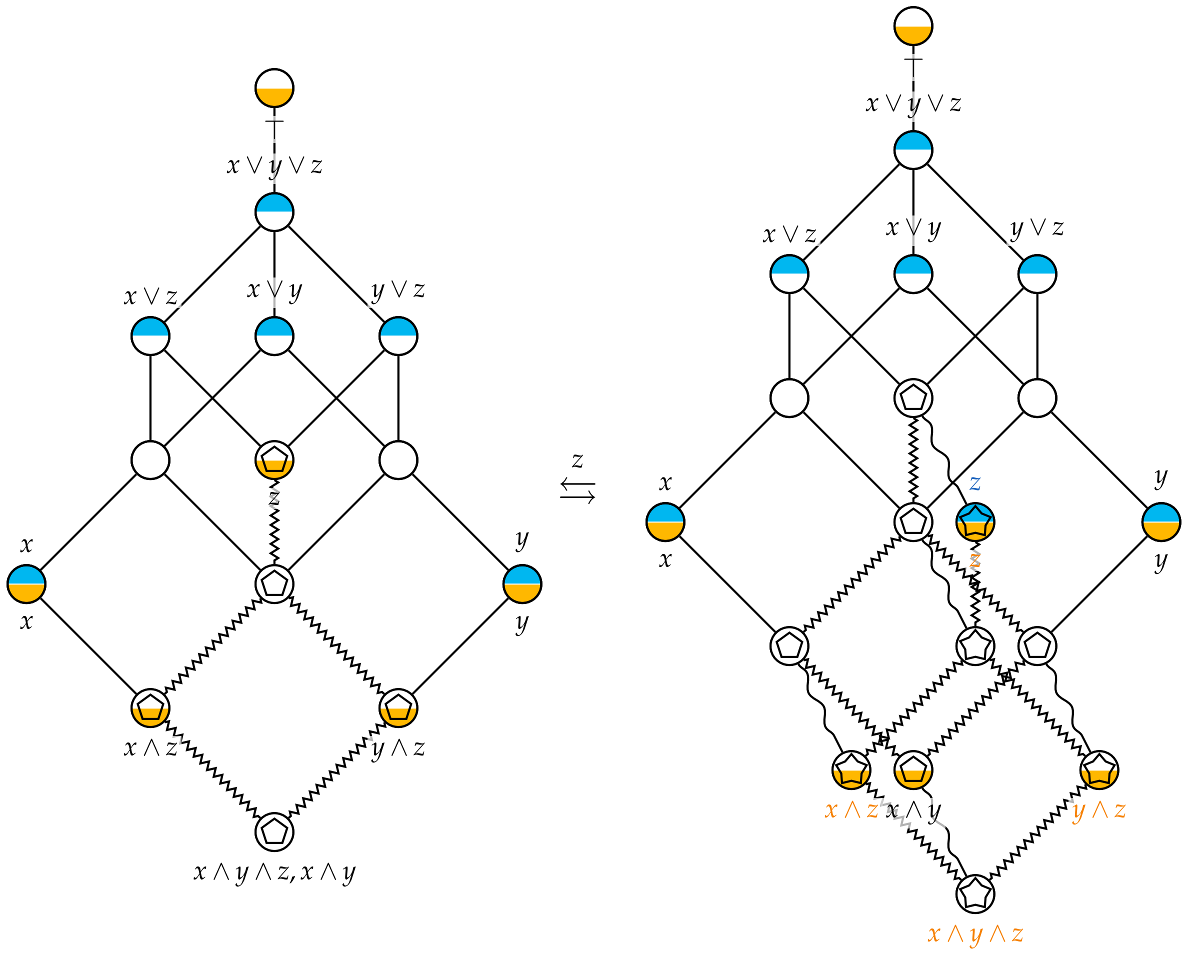
$$\forall_{\substack{(A,B)\in\mathfrak{B}^{\mathbf{C}}_{\mathsf{var}}(\mathbf{K})\\(C,D)\in\mathfrak{B}^{\mathbf{C}}_{\mathsf{gen}}(\mathbf{K})}} \mathsf{var}_{\mathbf{C}}(A,B) \prec_{\mathsf{v:n}} \mathsf{new}_{\mathbf{C}}(C,D) \Leftrightarrow \begin{cases} (A,B) < (C,D) \text{ and} \\ \nexists_{(X,Y)\in\mathfrak{B}^{\mathbf{C}}_{\mathsf{gen}}(\mathbf{K})\dot\cup\mathfrak{B}^{\mathbf{C}}_{\mathsf{var}}(\mathbf{K})} \quad (A,B) < (X,Y) < (C,D) \end{cases}$$

Furthermore, to have clear diagram structures, the reduciblity of the attributes and objects used as seeds must be updated. The reducibility of attributes can be updated via the following results.

(I) Each $\mathbf{K}$-reducible attribute is also $\mathbf{K}|\mathbf{C}$-reducible. A $\mathbf{K}$-irreducible attribute m is $\mathbf{K}|\mathbf{C}$-reducible, iff $\mu_{\mathbf{K}}(m) \in \mathfrak{B}^{\mathbf{C}}_{\mathsf{var}}(\mathbf{K})$ and $(\mu_{\mathbf{K}}(m))^* \in \mathfrak{B}^{\mathbf{C}}_{\mathsf{o}\neg\mathsf{g}}(\mathbf{K})$, and furthermore at least one superconcept of $(\mu_{\mathbf{K}}(m))^*$ is a generator concept.

(II) Each $\mathbf{K}|\mathbf{C}$-irreducible attribute from $\mathbf{K}$ is also $\mathbf{K}$-irreducible. A $\mathbf{K}|\mathbf{C}$-reducible attribute $m \in M$ is $\mathbf{K}$-irreducible, iff $\mu_{\mathbf{K}|\mathbf{C}}(m) \in \mathfrak{B}_{\mathsf{var}}(\mathbf{K}|\mathbf{C})$ has exactly one old upper neighbor ω and overthis only new upper neighbors, whose generators are superconcepts of ω.

Also it is possible to update object reducibility and even the (up) arrows as well. However, this is more complex, and is shown in my thesis.



Within the framework of my thesis, a Java package FCAFox for formal concept analysis has been written. The above presented results were used to formulate an algorithm iFox for the update task and was implemented and integrated into the CUBIST prototype at SAP.